Investing in Retirement with Floor and Upside Utility

Thank you for attending my presentation on investing in retirement with a floor and upside utility function.

Disclaimer

- Simulated not actual results
- Actual results will vary
- Investments may lose value
- Not peer reviewed
- Patent pending

Before we begin I would like to start with a brief disclaimer, which you should read.

Overview

- Floor and upside utility
- Computing the optimal strategy
- Optimal strategy without SPIAs
- Optimal strategy with SPIAs

Let’s begin with an overview of what I am going to cover.

First I will explain what a utility function is, why it is important, and what floor and upside utility functions look like.

Then using a floor and upside utility function I will cover how to compute the optimal investment strategy despite the uncertainties that exist.

And finally, the fun part of the talk. We will explore the optimal investment strategies both with and without SPIAs.

Floor and Upside Utility

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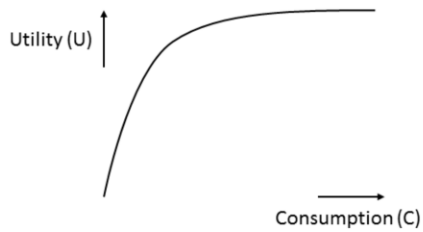
I will begin by explaining the concept of a utility function, which is a concept borrowed from economics.

This is the most mathematical section of the presentation, so I hope you will bear with me.

Utility

- Personal value placed on different levels of consumption



- As consumption increases utility increases at a slower and slower rate

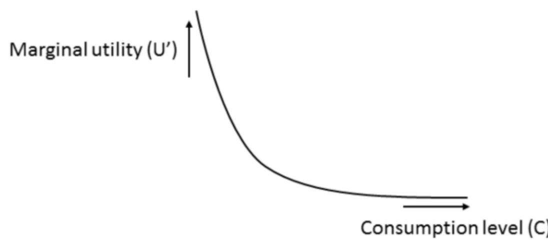
Utility, U, is essentially just a number associated with consuming different amounts of wealth. It represents a personal value for how happy consuming a given amount makes you. The more you consume the higher the utility.

However, as you consume more the utility increases at a slower and slower rate. A dollar when you are wealthy isn’t worth as much as a dollar when you are poor.

I will only consider annual utility. That is the utility associated with a given annual amount of consumption.

Marginal Utility

- Increase in utility for an incremental dollar of consumption



- As consumption increases marginal utility decreases

The decrease in the rate of change of utility can be formalized using the concept of marginal utility, U’.

Marginal utility is the increase in utility that results from an incremental dollar of consumption.

As the consumption level increases the increase in utility for an incremental dollar of consumption decreases.

Marginal utility may decrease at different rates for different individuals, or the rate of decrease may differ at different levels of consumption.

Risk Aversion

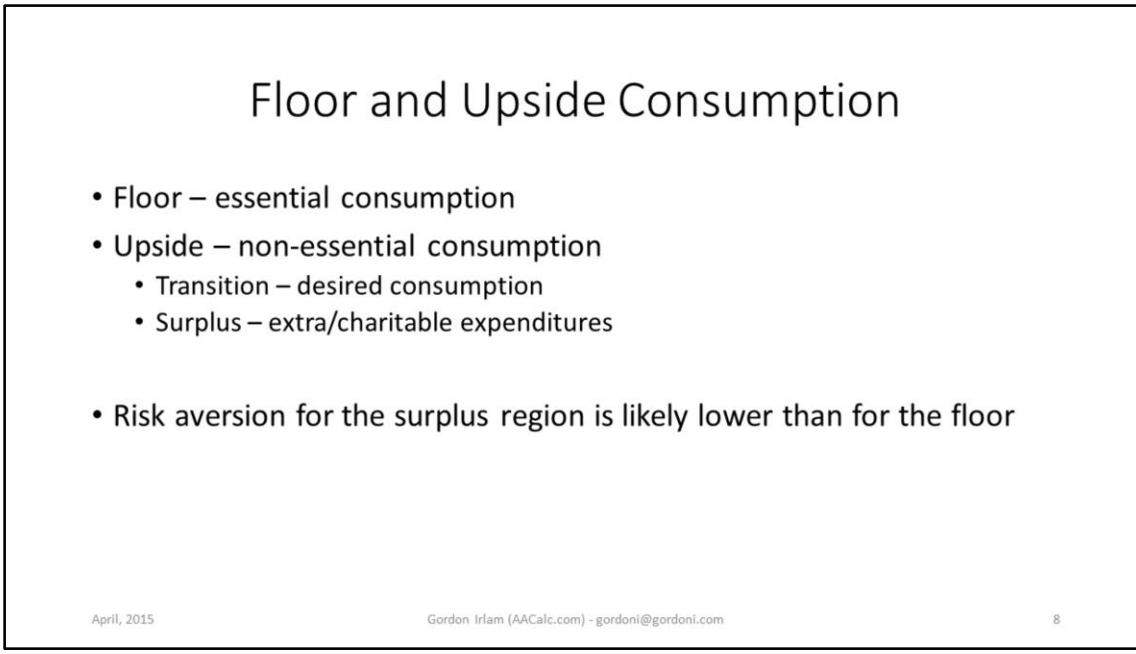
- Rate at which marginal utility decreases as consumption increases
- Can be described using the coefficient of relative risk aversion (RRA)

- High coefficient e.g. 5 → highly risk averse → U’ decreases rapidly → achieving lesser levels of consumption is very important
- Low coefficient e.g. 1 → low risk aversion → U’ decreases slowly → achieving higher levels of consumption is important

The rate at which marginal utility decreases as consumption increases is known as the risk aversion. Most commonly it is described using the coefficient of relative risk aversion, which has a precise mathematical definition that I am going to skip over.

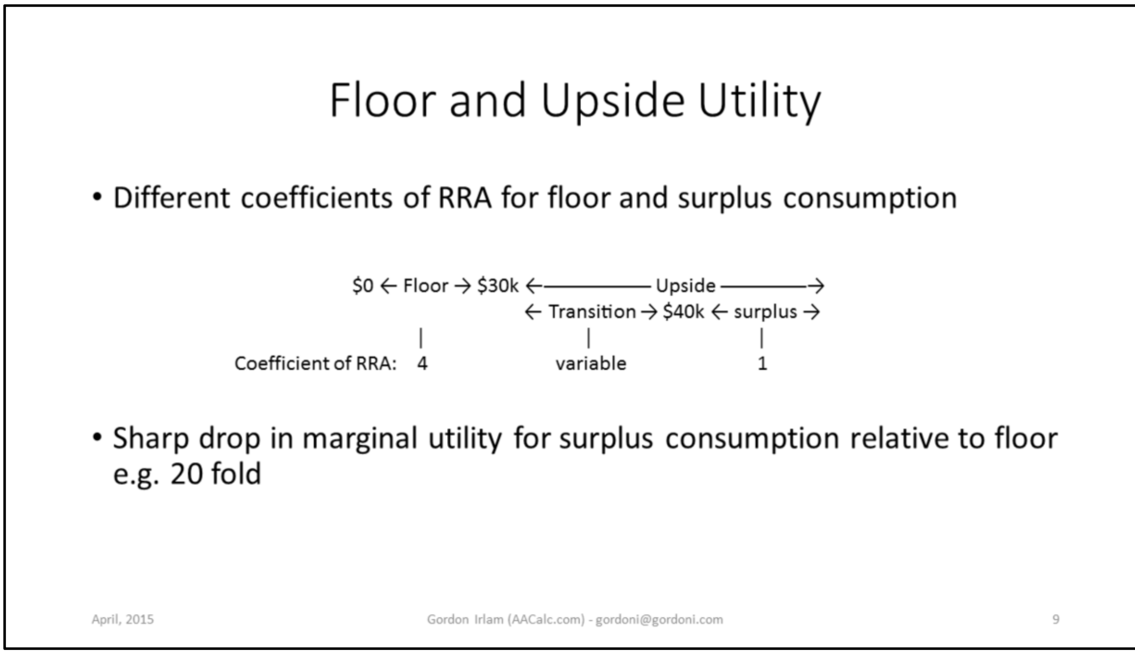
What you need to know is if you have a high coefficient of relative risk aversion, such as 5, you are highly risk averse, marginal utility decreases rapidly, and you care little about achieving high levels of consumption. Soldifying lower levels of consumption is what counts.

Conversely, if you have a low coefficient of relative risk aversion, such as 1, you have a low aversion to risk, marginal utility decreases slowly, and achieving high levels of consumption is almost as important as achieving lower levels of consumption.



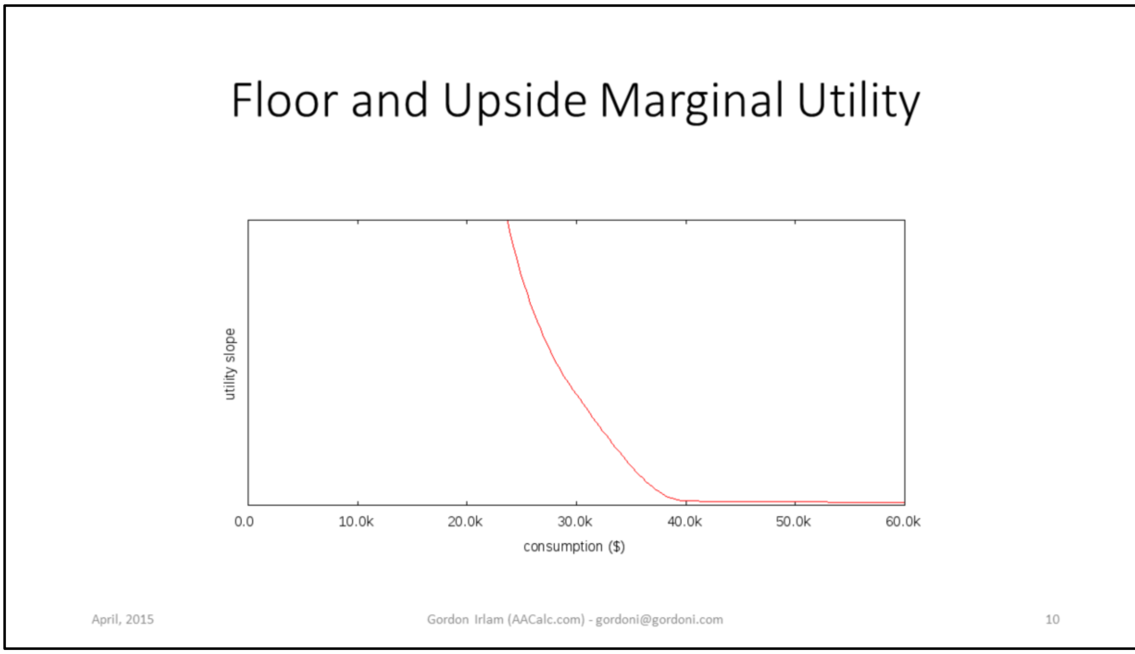
I divide consumption into two parts. Floor consumption for essential expenditures, and upside consumption for everything else. In my model upside itself is sub-divided into two sub-parts. The surplus region which covers spending when you have so much money you don't know what to do with it. And a transition region that joins the floor to the surplus region and incorporates desired spending on items such as vacations.

Risk aversion for the surplus region is probably going to be substantially lower than it is for the floor. An incremental dollar is a lot more valuable at $10k than at $20k of consumption. But an incremental dollar of charitable donations at $100k might only be slightly more valuable than an incremental dollar of charitable donations at $200k.



With all that out of the way, we can finally get to what I am calling a floor and upside utility function. It is a utility function that has different coefficients of relative risk aversion in the floor and surplus regions. So the first $30k of consumption is highly risk averse and has a coefficient of relative risk aversion of 4. While above $40k we are only weakly risk averse and the coefficient of relative risk aversion is 1. In between $30k and $40k is the transition region that joins the floor and surplus.

Besides simply specifying the coefficients of relative risk aversion we also need to specify relative marginal utilities. We chose to specify surplus expenditures are 20 times less valuable than floor expenditures, indicating only a weak preference to be engaged in charitable expenditures. We will explore the effect of reducing this factor later.



This plot shows the marginal utility, also called the slope of the utility function, for different levels of consumption.

Below $25k the marginal utility is so high that it doesn't fit on the graph. This is the result of our using a high coefficient of relative risk aversion.

Above $40k the marginal utility is low, but it decreases slowly due to the low coefficient of relative risk aversion.

Why Utility Matters

- All consumption isn't equal
- Floor consumption is more important than surplus consumption
- Utility lets us express by how much

- Investment goal should be to maximize expected utility (not consumption and definitely not wealth)

This completes our exploration of utility.

It is worth pausing for a moment to understand why utility matters. Utility matters because it provides a rigorous way for us to express the concept that floor consumption is more important than surplus consumption.

When investing the goal should be to maximize expected utility, not consumption. An example will make this clear. Would you rather consume a guaranteed $30k for a year, or a 50/50 chance of consuming either $10k or $90k? $10k is way too little to live on. And $90k would add little to your happiness beyond what $30k provides. The certain outcome has a lower expected consumption value, but it has a higher expected utility, making it preferable.

Computing the Optimal Strategy

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The investment problem is deciding when and how much of your assets to consume and how to invest the remainder.

The approach I take to solving this problem differs from the most common approach to retirement planning, so it is worth reviewing the most common approach.

A Common Approach to Retirement Planning

- Trial and error
  - Pick a strategy
  - Test to see how well it does e.g. Monte Carlo simulation
  - Modify the strategy
  - Test it again
  - Repeat

The most common approach to retirement planning involves Monte Carlo simulation.

You come up with a strategy, and then you test it to see how well it performs using either historical or projected future data.

Based on the results you tweak the strategy and again see how well it performs.

This approach has its advantages, it is easy to understand and implement, but it also has its problems.

Problems with Trial and Error

- Making changes and re-running is time consuming
- Impossible to test every possible strategy
  - 10 asset allocation possibilities each year
  - 10 consumption possibilities each year
  - 30 year investment horizon
  - 100^{30} Monte Carlo simulations to perform
- Will never know if the final strategy is optimal

Making changes to the strategy in an attempt to improve performance can be time consuming.

But the big problem is the search space being searched over for an optimal solution is huge. E.g. if we have 10 asset allocation possibilities, and 10 consumption possibilities each year, (these are low estimates of the number of possibilities to be explored), and a 30 year investment horizon, then there are 100 to the 30, or 10 to the 60 different possible Monte Carlo simulations to run.

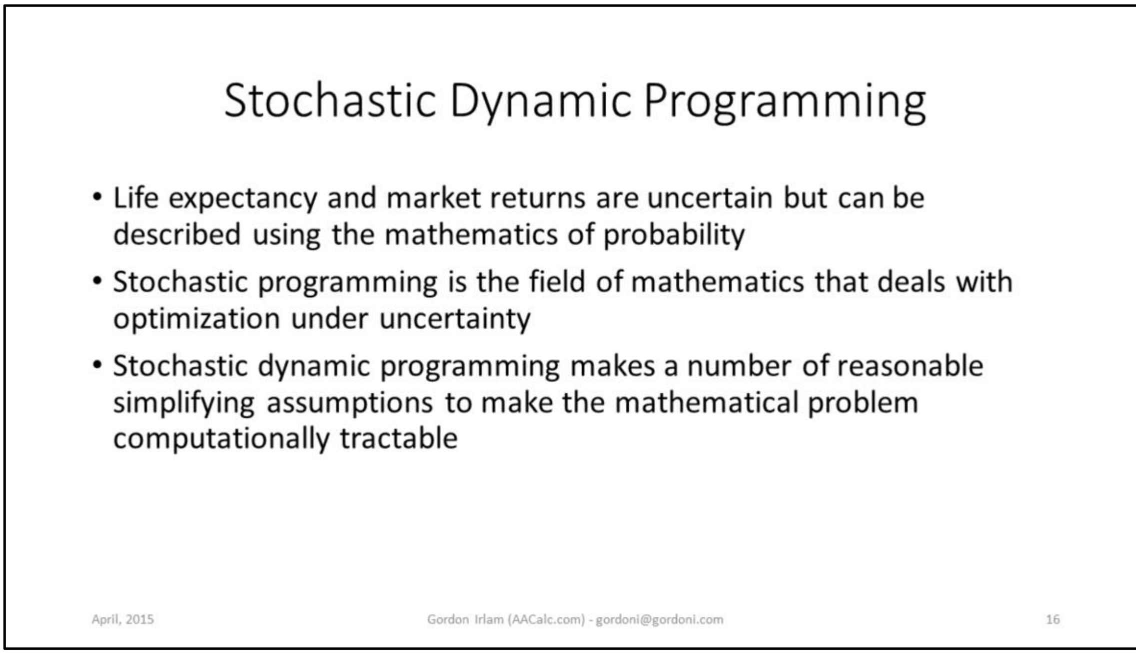
With trial and error it is going to be very difficult to find the optimal strategy, and even if you encounter it, you have no way of knowing that the strategy is optimal.

A Better Approach

- Finding the best strategy is a mathematical optimization problem
- Express it mathematically
- Directly compute the optimal strategy

I think a better approach is to reformulate the problem as an optimization problem using the language of mathematics, and to solve it directly.

This delivers the optimal solution, and you also know that it is optimal.



Stochastic means random or uncertain, like the randomness associated with the age you live to, or market returns.

The word programming predates the computer, and means coming up with a program, a timetable, or a schedule, which is what we need. A step by step schedule of how much to consume, how to invest, and when to do so.

Dynamic programming involves breaking a complex problem down into a collection of simpler sub-problems. In our case it means the investment strategy at age N with assets A is independent of how we arrived at N and A. We can compute the optimal strategy without having to consider the past.

So we compute the optimal strategy using stochastic dynamic programming.

What is Modeled

- Asset allocations that vary over time
- SPIA purchase decisions that vary over time
- Consumption that varies over time
- Variable life expectancy
- Stock and bond returns from a known distribution
- Non-normal stock and bond distributions
- Correlated stock and bond returns
- Inflation rates from a known distribution

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Using stochastic dynamic programming provides a lot of flexibility.

We can, and will, be concurrently optimizing over all of the items listed.

Take the first item listed, asset allocation. In the first year we might consider a 50/50 asset allocation, a 40/60 allocation in the second year, and a 70/30 allocation in the third year.

Similarly, SPIA purchase decisions and consumption may vary over time.

To try and find the optimal strategy using trial and error would be very challenging.

Life expectancy is uncertain. I optimize taking this into account.

Similarly, stock and bond returns are uncertain, need not come from a normal distribution, and may be correlated with each other.

Finally, the inflation rate is uncertain.

Stochastic dynamic programming can handle all of these factors.

What is Not Modeled

- Autocorrelated return distributions
  - Momentum
  - Mean reversion
- Variable volatility
- Variable correlations
- Sticky inflation rates
- Annuity prices moving with the interest rate environment

There are some things which I don't model.

Autocorrelations, of periods of 1 year or more, for aggregate stock and bond markets, not individual stocks, appear relatively weak, so this isn't a major loss.

Volatility and asset class correlations both change over time, so it might be possible to do better than our optimal solution by taking advantage of this.

If you have low inflation one year, you are likely to have low inflation the next year, but my model doesn't incorporate this.

Lastly my model uses a fixed yield curve for computing annuity prices, and has no notion of the yield curve varying over the business cycle.

What Does it Take

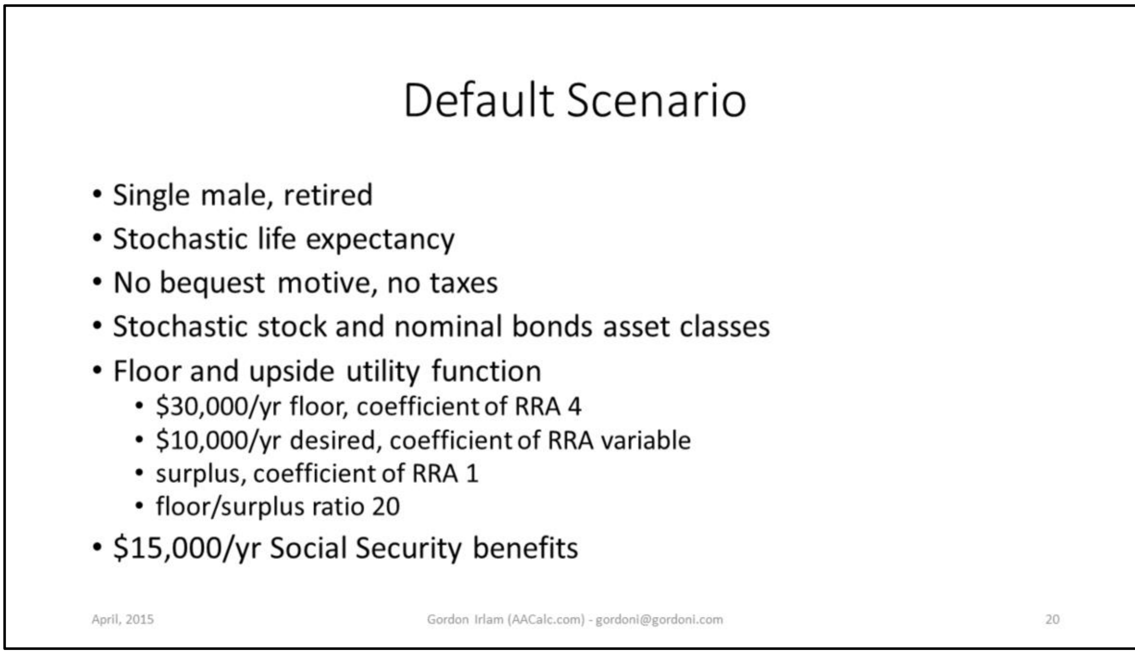
- Without SPIAs
  - 1 second
- With SPIAs
  - 3 hours

- Website to compute optimal strategy without SPIAs publicly available
- Optimal strategy with SPIAs is too expensive
  - Have to mentally project described results

Without SPIAs I am essentially solving a two dimensional problem. Find the optimal strategy as a function of age and portfolio size. This is fast. It takes around 1 second.

With SPIAs there is an extra dimension over which to calculate the optimal strategy, the SPIA income currently being received. This is slow. Around 3 hours.

I have created a website and made it freely available to compute the optimal strategy without SPIAs. If you use that website you will notice the compute time is closer to 1 minute than 1 second. This is because it is running on a relatively slow computer, and in addition most of the time is being spent comparing how well the strategy performs in comparison to other strategies such as age in bonds/4% rule. Stochastic dynamic programming itself is fast.



The default scenario I will be considering is.

I will only consider a retired individual. Adding pre-retirement income complicates the analysis more than it offers any new insights. For pedagogical purposes I will sometimes show graphs with a retired individual as young as 25.

Stochastic life expectancy means the individual will be dying at random in accordance with the Social Security Administration Actuarial Study 120 Life Table.

No bequest motive is important when we come to consider SPIAs. If there was a bequest motive SPIAs would be less attractive.

The stock and bond asset class returns are from Shiller but with stocks adjusted to reflect the average global returns over the past century, and bonds adjusted to reflected the US investment grade bond universe.

The floor and upside utility function is as seen earlier.

Lastly, and very importantly, the individual receives $15k per year of Social Security benefits. Social Security has bond like qualities, and so the presence of Social Security has a big impact on the remaining asset allocation for underfunded portfolios.

The Process

- Use stochastic dynamic programming to compute optimal strategy
- Represent the strategy using a series of age and portfolio size maps (one for each strategy component i.e. consumption amount map, asset allocation map, and SPIA purchase amount map)
- Perform Monte Carlo simulation that follows the maps to see how the strategy performs

The process I use has two steps.

First, compute the optimal strategy using stochastic dynamic programming. The output of this step is a series of maps that describe the optimal strategy as a function of age and portfolio size. Or for SPIAs also a function of an additional dimension, the SPIA income amount.

Then, the second step is to perform a Monte Carlo simulation using the maps to see how the strategy performs.

It is important that you understand both of these steps as I will be inter-mixing slides from both of them. I compute the strategy, then I evaluate how well it performs.

Optimal Strategy without SPIAs

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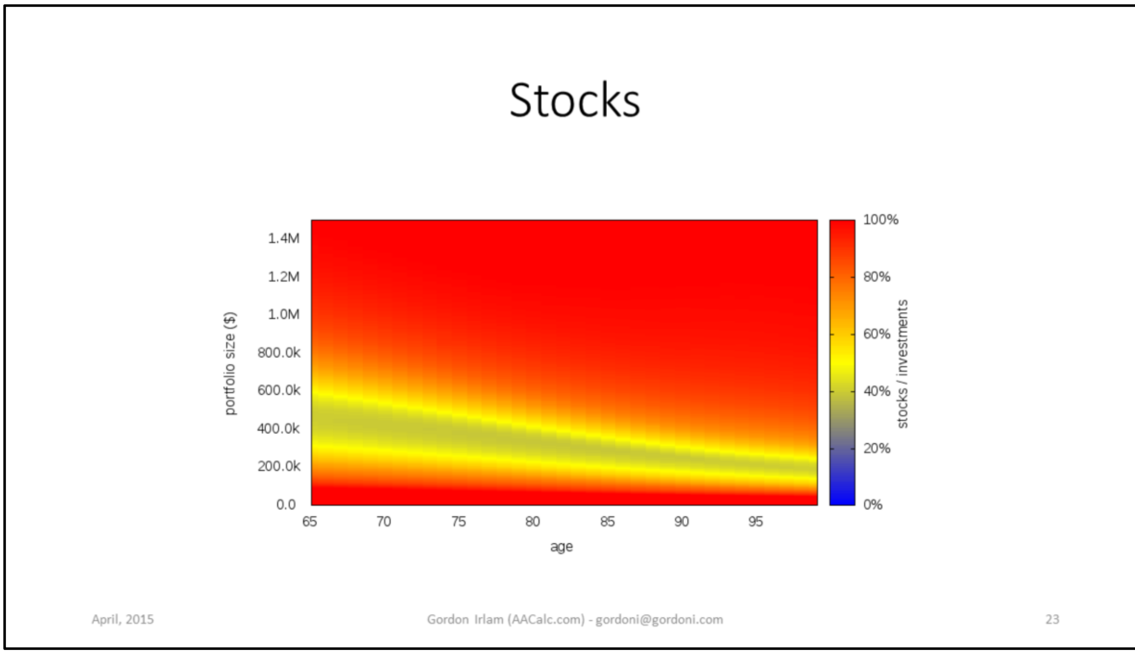
With all the preliminaries out of the way, now we get to the fun part of the presentation. What does this approach tell us about optimal retirement investment strategies.

First I will consider the optimal strategy without SPIAs. Just stocks and bonds.

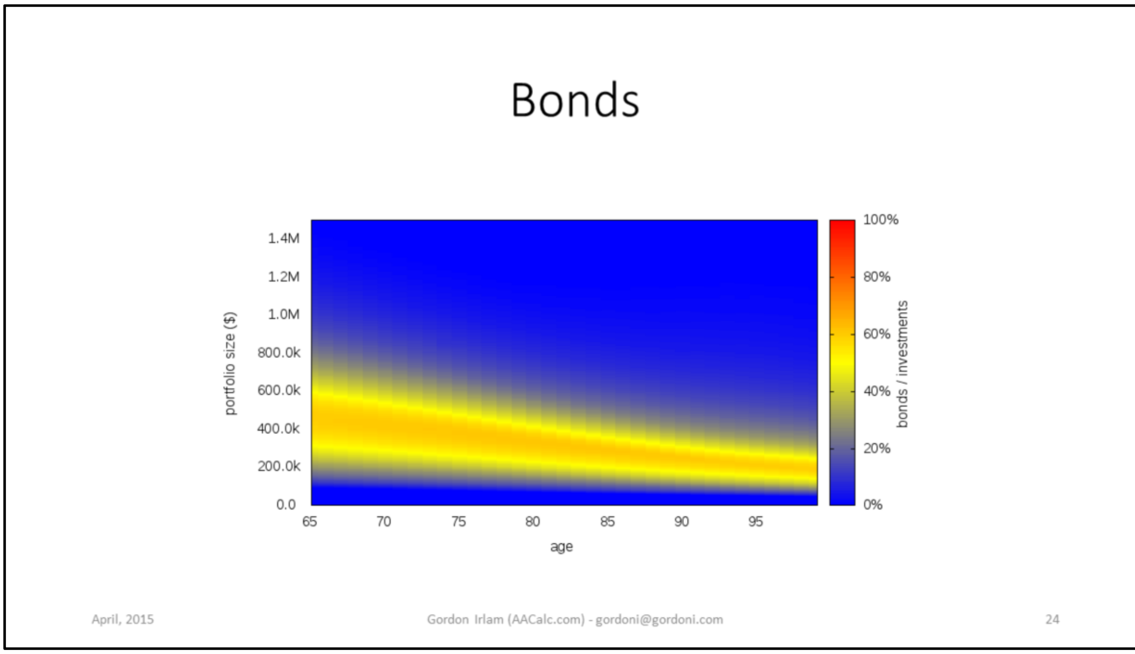


This graph shown is called a heatmap, and there are going to be a lot of heatmaps, so it is important that you know how to read them. This is a plot of the optimal stock asset allocation strategy. It shows the optimal strategy as a function of age and portfolio size. Along the bottom we have age, and going up on the left we have portfolio size. Let's pick age 70, and a portfolio size of $400k. If we look on the graph where they intersect we have a dirty yellow color. Now if you turn to the key on the right hand side, you will see that the dirty yellow color roughly corresponds to a stocks / investments of 40%. In other words if you are 70 and have a $400k portfolio your optimal asset allocation is 40/60. The key will always range from blue, 0%, to yellow, 50%, up to red, 100%.

What this graph says is if you are underfunded you should be investing in stocks, and/or finding ways to reduce your required consumption. The reason for recommending stocks to the underfunded is Social Security already provides them a large bond like investment component.

The yellow–dirty-yellow band is essentially if you are on track. Your portfolio should be more or less balanced.

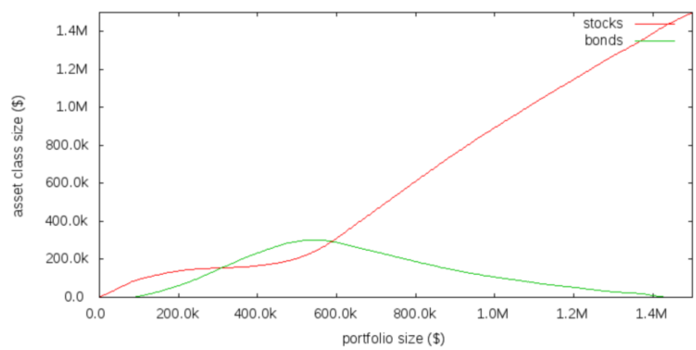
Above this band is for the overfunded. They are likely to be consuming in the surplus region. As a result of the low risk aversion in this region, they should be 100% stocks for maximum return. And only if reality turns out to fall below expectations should they adopt the more defensive posture of the yellow band.



The graph for bonds is simply the mirror image of the graph for stocks. Where stocks are high bond are low.

Next we are going to take a vertical slice though the two asset allocation graphs at age 65, and look at the recommendations in more detail.

Stocks and Bonds at Age 65



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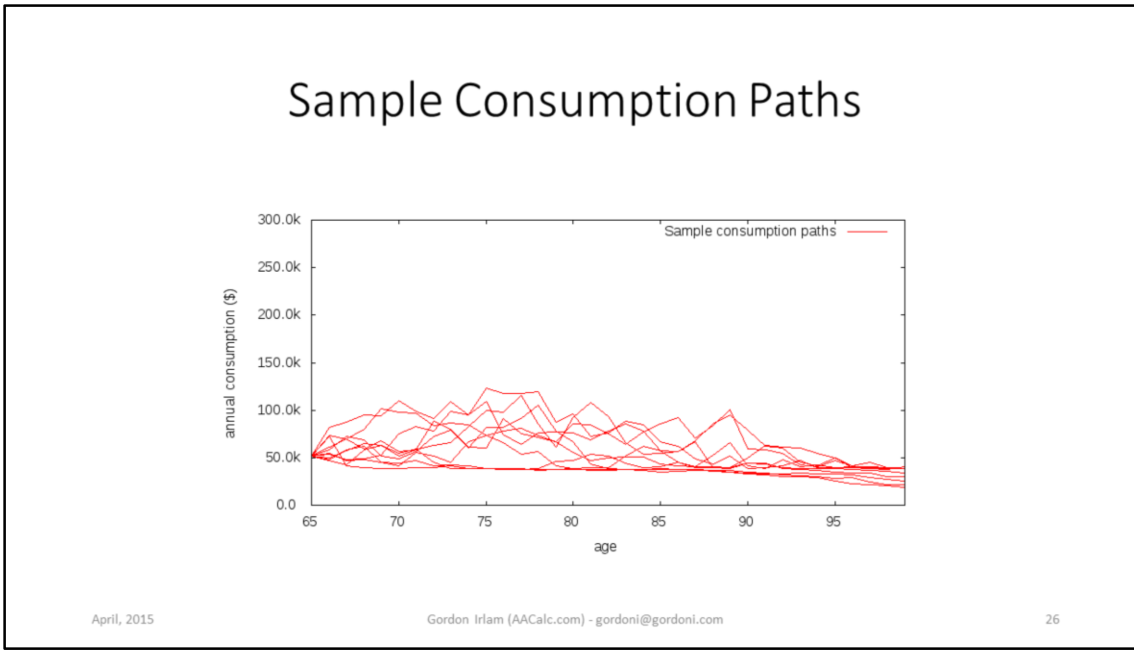
Here we are just looking at the recommendations for age 65.

The x-axis is portfolio size, and the y-axis is asset class size. The red line is for stocks, and the green line is for bonds.

Starting from zero, as you can see when we first start adding to the portfolio, we add exclusively to stocks. But once we reach about $200k, things plateau out, we stop adding to stocks, and we add almost exclusively to bonds. This continues until about $500k, at which point stocks take off again, and bonds start to fall, until at $1.4m there are no bonds, just stocks.

It is interesting to compare these recommendations with the traditional recommendation of building the floor using low risk assets, then building the upside. Here we find first build with stocks to offset Social Security, then build the remainder of the floor using low risk assets, and then finally build the upside, and slowly dismantle the low risk floor if you are wealthy enough.



Now I switch to Monte Carlo simulation

I haven't shown the optimal consumption map. It is fairly boring. The more you have, and the older you are, the more you should consume.

The graph here shows ten Monte Carlo paths following the optimal asset allocation and consumption maps for an individual that had $750k at age 65. Consumption is on the y-axis.

The important thing to notice is that base consumption is very stable, at around $40k per year, while the surplus consumption is far more volatile. This is what I requested when I specified the utility function to optimize using stochastic dynamic programming. It is nice to see it born out.

Consumption starts to dip around age 95. This reflects a strategic decision of the optimization machinery that the individual is unlikely to be around by then, so it doesn't make sense to set aside a large amount of assets for the possibility. Even when consumption does dip at this age it does so gradually, not catastrophically.

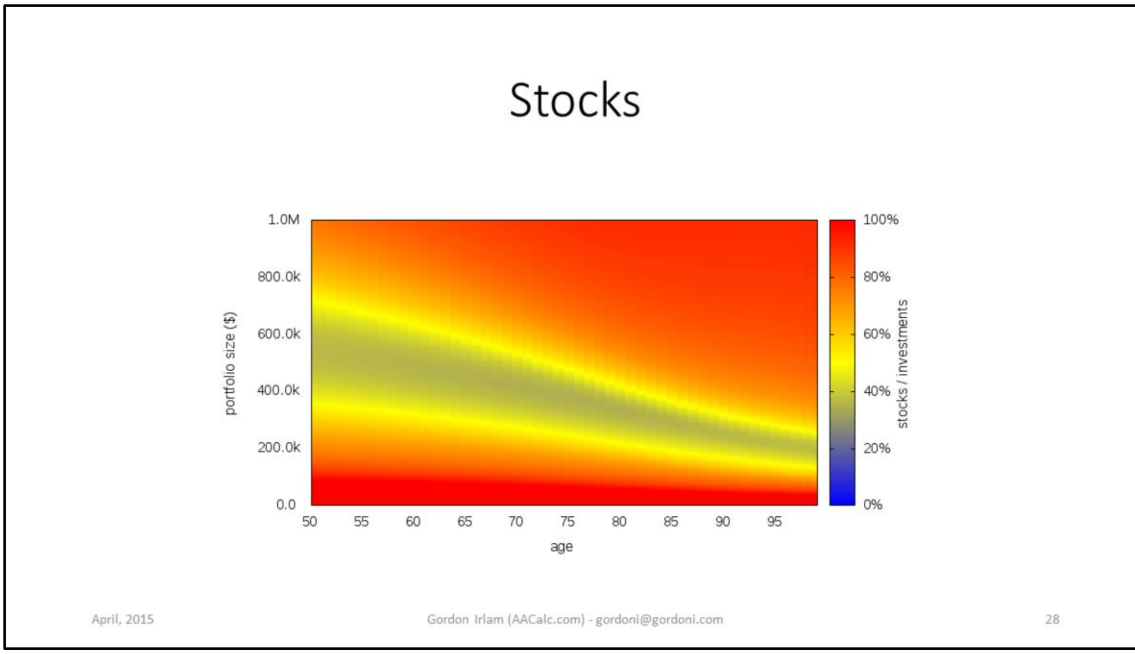
Adding TIPS

- Constant maturity TIPS have lower real volatility than nominal bonds
- And slightly lower expected returns

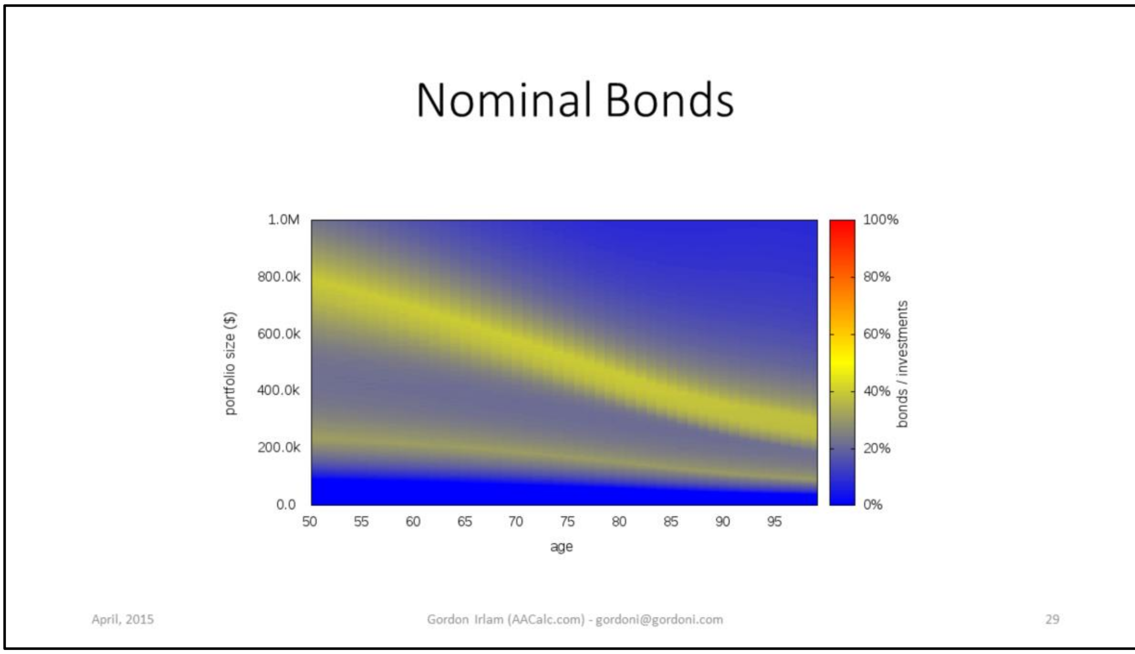
Now we will look at what happens when we add TIPS to the stock/bond asset mix.

It should be stressed that this is not a liability matching portfolio, but constant maturity TIPS, similar to a TIPS mutual fund.

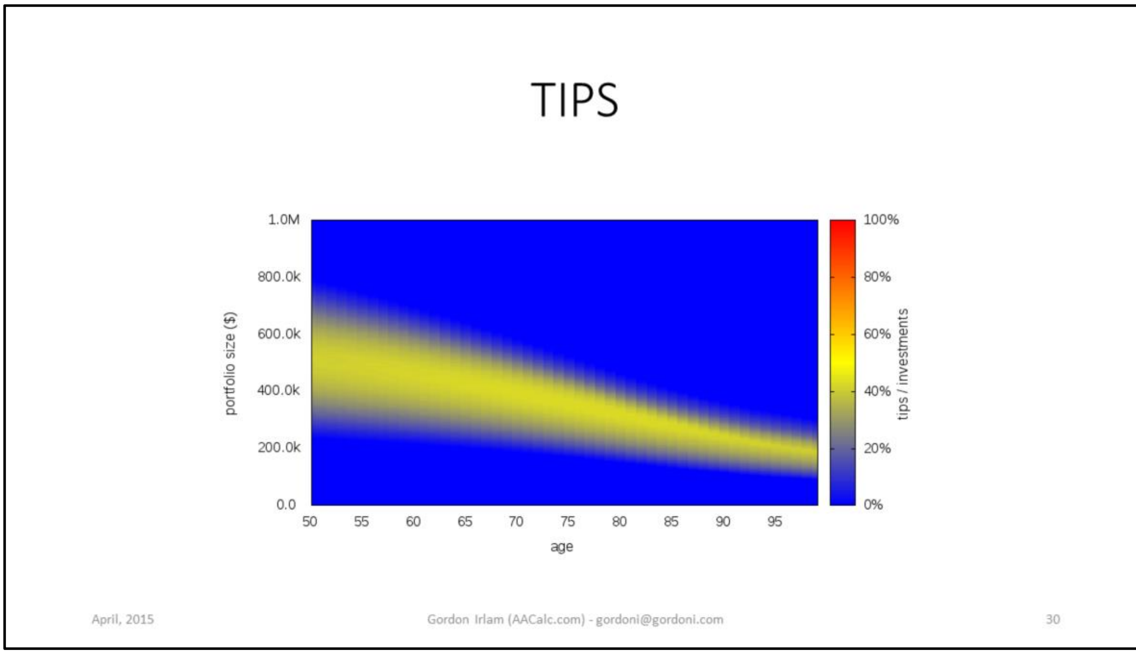
Does the greater real stability of TIPS make them preferable to nominal bonds? We will find out.



The plot for stocks looks similar to before. Only the x-axis has been changed to start at age 50, and I have zoomed in slightly on the y-axis. This is for compatibility with what is to come.



The graph for nominal bonds though has changed significantly. The center of the yellow band is missing.



And the missing center of the bond band shows up as TIPS.

In other words TIPS are recommended in the region where bond holdings peak, and presumably stability is most desired. At the edges of the bond band where there are more stocks it is a mixture of nominal bonds and stocks that rules the day.

Optimal strategy with SPIAs

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Now it is time to add SPIAs to the asset mix. Stocks, bonds, and SPIAs.

We will primarily focus on nominal SPIAs, but we will also consider real, or inflation indexed SPIAs. Inflation indexed SPIAs are linked to the CPI, and are not the same as inflation adjusted SPIAs which may receive a fixed annual inflation adjustment, such as 2%.

SPIA Pricing

- Mortality and bond yield curve determines SPIA cost
- Mortality is allowed to vary over time (cohort improvements)
- Simplification: yield curve is static
  - Acceptable because bond returns are random
  - No notion of bonds being cheap or expensive

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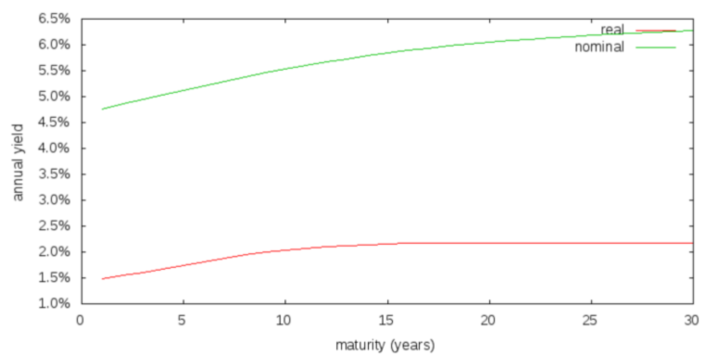
We need to spend a while discussing SPIA pricing. We live in an unusual interest rate environment, and if we were to blindly use it in computing SPIA prices we would get erroneous results. Instead we need to reverse engineer SPIA prices computing the money's worth ratios used, and then use those ratios to recreate the would be SPIA prices in a more normal interest rate regime.

SPIA prices can be considered to be made up of three components: mortality, the interest rate or yield curve, and profit and overhead determined by the money's worth ratio.

For mortality I use the Society of Actuaries 2012 Individual Annuity Mortality Basic table. I adjust mortality to take into account cohort improvements. I also adjust the table to take into account the fact that recent annuitants are likely to have a lower mortality than annuitants who have held their policy for a number of years.

Due to limitations of stochastic dynamic programming we use a fixed yield curve for the interest rate.

Yield Curve

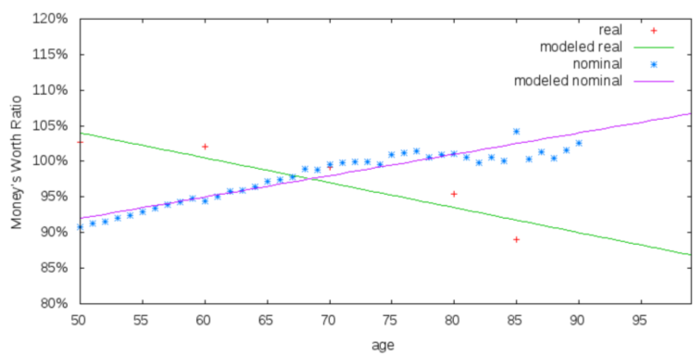


The shape of the zero coupon bond yield curve is shown in this figure. The red line is the real interest rate, and the green line is the nominal interest rate.

The real interest rate was obtained by averaging the Treasury TIPS rates. The nominal interest rate came from averaging the High Quality Markets (AAA, AA, and A) bond rates.

The height of the yield curves is adjusted to reflect the average interest rates over the period of study.

Money's Worth Ratio (MWR)



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I recorded the best available SPIA quotes for real and nominal SPIAs on a particular date, as well as the corresponding yield curves.

The implied money's worth ratios are shown by points, while the lines are the money's worth ratios we used to compute SPIA prices. Using smooth lines rather than the actual quotes results in smooth annuitization graphs, instead of noisy graphs that depend on the vagaries of the quotes.

Money's worth ratios in excess of 100% shouldn't occur. That they do, reflects either an inaccuracy in the mortality and yield curve assumptions, or less likely, a market pricing error. In any event it is gratifying to see money's worth ratios are almost universally above 90%, indicating good value for the consumer.

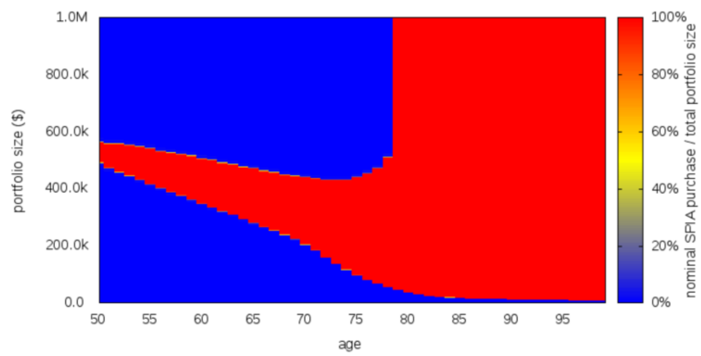
The nominal SPIA money's worth ratio is upward sloping. This could occur if the interest rate insurance companies obtain was less than that of the High Quality Markets, such as by having more AAA and AA bonds.

The real SPIA money's worth ratio is downward sloping. This is harder to explain, but it might be the case that the insurance companies are able to obtain a higher rate than the TIPS yield curve, perhaps by using corporate inflation linked securities.

In any case these slopes will likely mean a preference for real SPIAs at young ages, and nominal SPIAs at advanced ages.

This graph shows the price of SPIAs in cost per dollar of income per year. The main take away is that the difference between the actual quotes, the points, and the modeled prices, the lines, is very small.

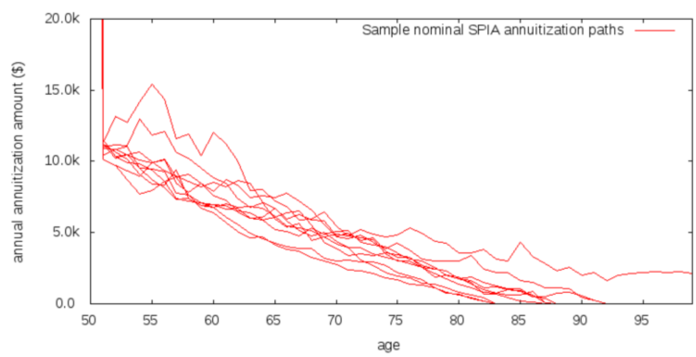
Complete Annuitization



This graph shows when to annuitize with nominal SPIAs when annuitizing involves annuitizing 100% of your portfolio.

Roughly speaking, you should always annuitize by age 80, and there is a tapered band where it makes sense to completely annuitize earlier. This band probably corresponds to the region where it is more valuable to lock in floor and transition consumption than it is to try and achieve significant surplus consumption.

Post Annuitization Nominal SPIA Purchases



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This graph shows continued annuitization activity for an individual that completely annuitized their $500k portfolio at age 50.

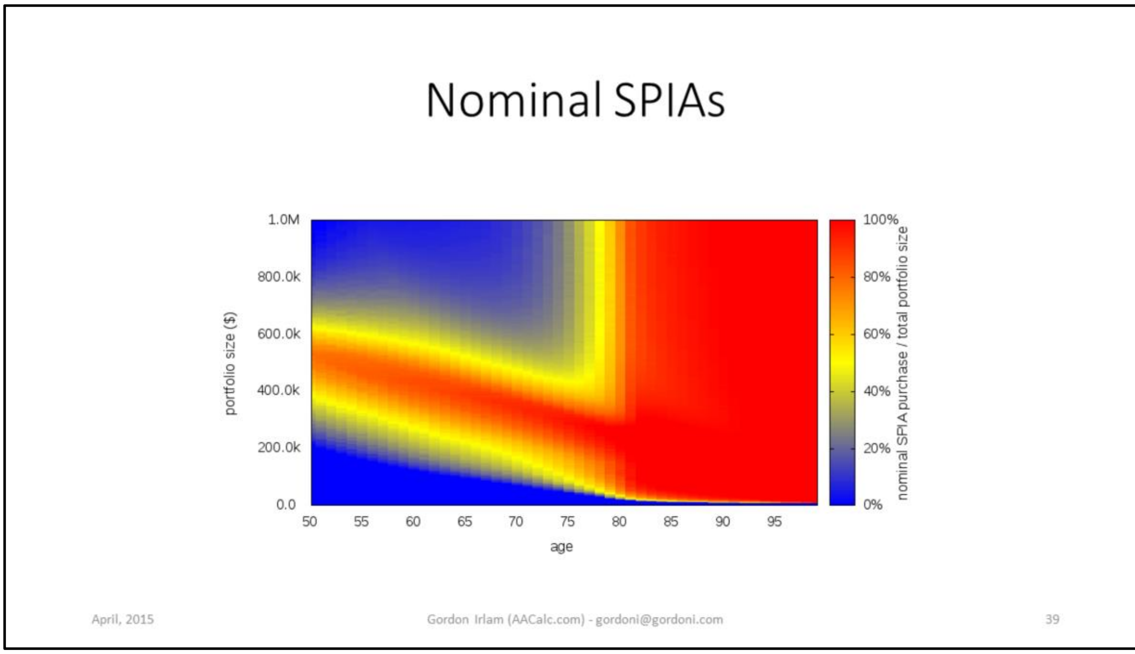
Nominal SPIAs gradually lose purchasing power due to inflation. To prevent this, rather than consuming all of the SPIA income, it is necessary to re-annuitize a portion of it. In the optimal strategy this occurs every year, in a more realistic strategy part of the SPIA income might be saved, and re-annuitized every 10 years. This re-annuitization no longer appears necessary after about age 85.

The difficulty of this, particularly computing the appropriate amount to annuitize, is one of the advantages of using real SPIAs.

Gradual Annuitization

- Annuitization doesn't have to be an all or none affair
- Take the other extreme
  - Each year can annuitize a portion of the non-annuitized assets
- Not intended as a real world strategy
- Intended to provide insight into simpler real world strategies
  - E.g. annuitize a portion of your assets early and annuitize the remainder later

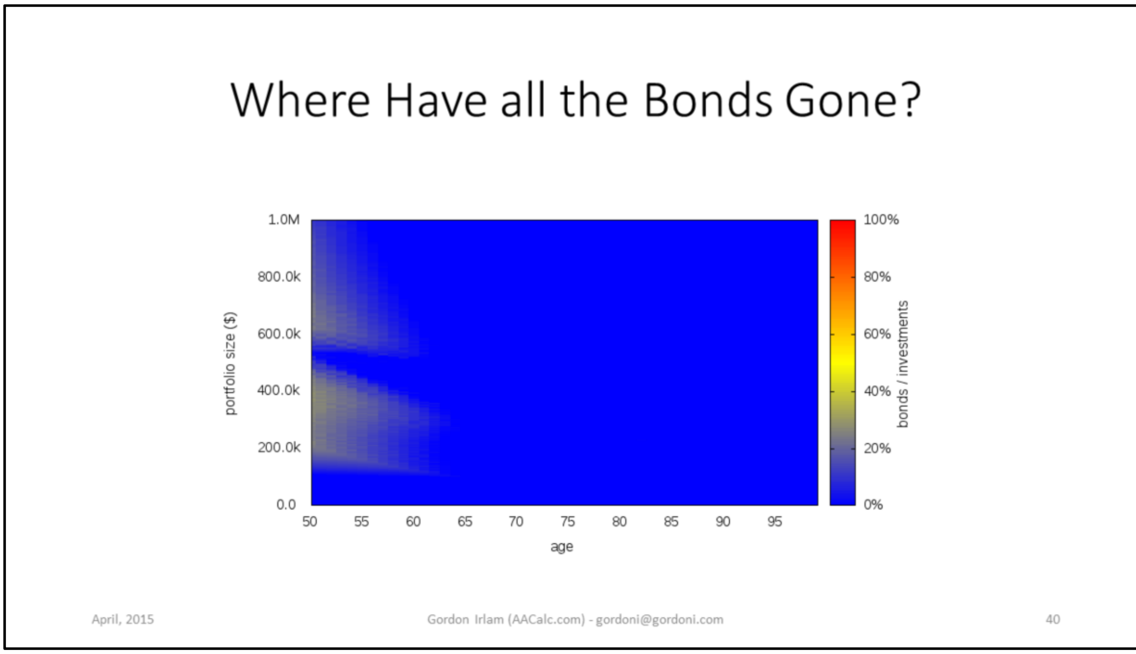
Now we switch to gradual annuitization, which involves computing the optimal strategy when you have the option of annuitizing part of your assets. It is unlikely anyone would want to annuitize a part of their assets every year, but by computing the optimal gradual annuitization strategy we can gain insight into a more reasonable strategy to implement.



This graph shows the optimal nominal SPIA annuitization fraction for an individual that currently doesn't hold any SPIAs. The SPIA annuitization fraction is, the amount of SPIAs to be purchased, divided by, the portfolio size prior to purchasing the SPIAs. The yellow region thus represents the region in which an individual without SPIAs would annuitize 50% of their portfolio.

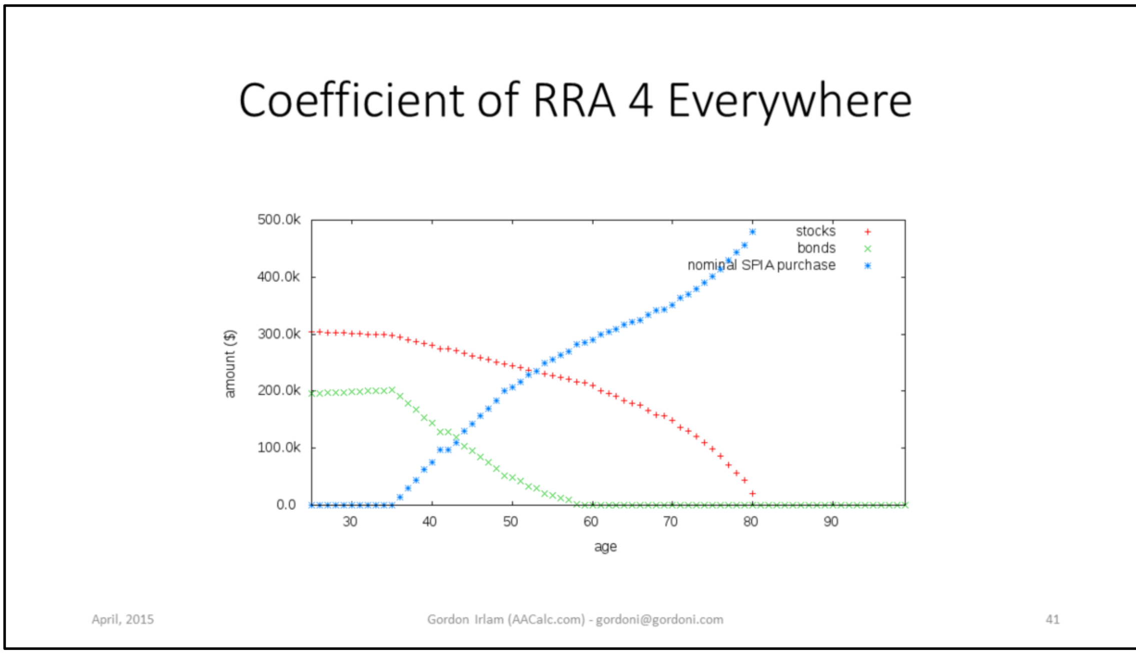
For an individual that already holds SPIAs, add the market value of those SPIAs to the investment portfolio size, and use this graph to determine the minimum amount of SPIAs he should presently hold. If he holds less, buy more. If he holds more, fine.

Note that heavy annuitization can occur early for particular portfolio sizes.



This slide shows the asset allocation, that is bonds divided by stocks plus bonds, for the nominal SPIA gradual annuitization case.

There are almost no bonds held beyond age 60. Nominal SPIAs have taken the place of bonds.



The previous graphs were complex because our utility function is complex.

To get a better handle on the replacement of bonds by SPIAs, and to find out what happens to stocks, we use a simple utility function in which the coefficient of relative risk aversion is 4 everywhere, rather than just in the floor region. And in order to see what is happening we consider retirement ages as young as 25.

The graph shows how $500k would be invested between stocks, bonds, and nominal SPIAs in this simplified case.

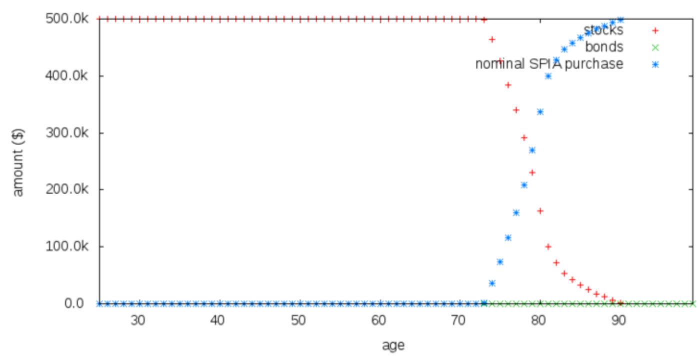
Initially no annuitization occurs, and the ratio of stocks to bonds is roughly a constant. This is a famous result of Samuelson and Merton that a constant relative risk aversion utility function implies a fixed asset allocation, independent of age and portfolio size.

Bonds get annuitized starting at around age 35, while stocks are primarily annuitized later.

Obviously then, stocks and bonds, are not annuitized at the same rate, and the ratio of stocks to bonds varies significantly. This is worth mentioning because it might be erroneously assumed that annuitization doesn't effect asset allocation. It does.

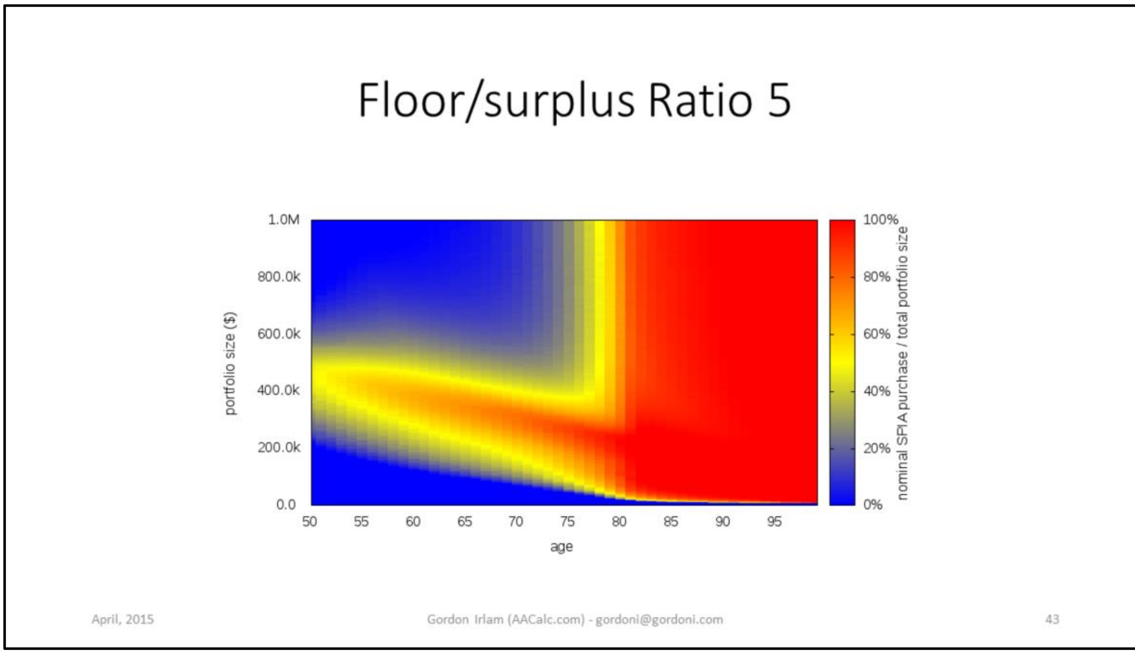
Note that the mid-point of stock annuitization, with half of the total stock holdings having been annuitized, is around age 70.

Coefficient of RRA 1 Everywhere

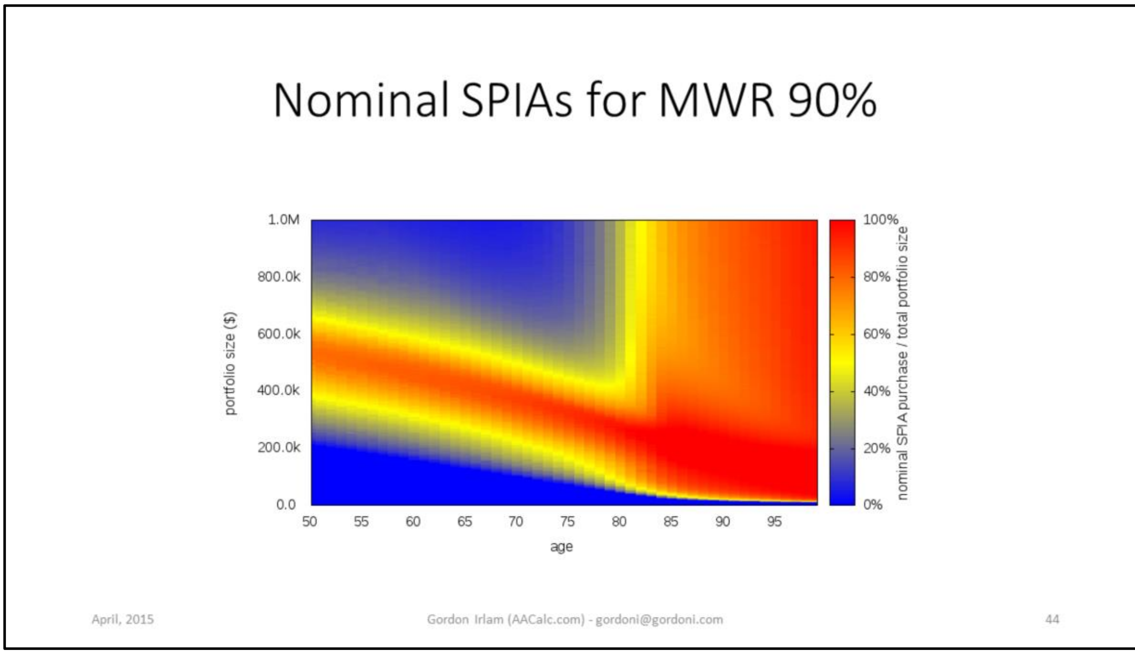


A coefficient of relative risk aversion of 1 tells a different story.

Here there are no bond holdings ever, and the mid-point of stock annuitization is around age 80. There is a huge difference between a coefficient of relative risk aversion of 4 and 1, but the mid-point for stock annuitization only moves by about 10 years.



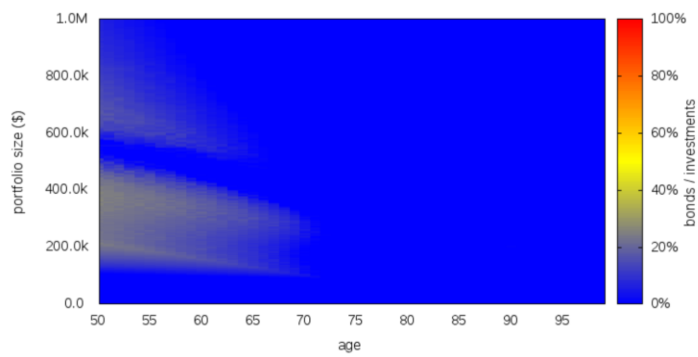
I had been using a floor/surplus marginal utility ratio of 20. When I reduce it to 5, reflecting a greater interest in charitable endeavors, there isn't a lot of difference in the optimal strategy. The intensity of maximal annuitization is reduced, but substantial annuitization remains. Complete or near complete annuitization might be less favored as a strategy, but it would appear SPIAs are still preferred over bonds.



You might remember that nominal SPIAs had an upward sloping money’s worth ratio. What effect might that have?

Reducing the money’s worth ratio to 90% everywhere, only results in small changes of style, not of substance.

Bonds for MWR 90%



Bonds are still replaced by SPIAs, but now the bonds aren't fully gone until around age 65 or 70.

The Bottom Line

- Improvement in portfolio consumption

Starting age	50	65	80
Starting portfolio size	$500k	$400k	$300k
Nominal SPIAs	13%	21%	53%
Real SPIAs	20%	21%	41%

- Difference between nominal and real SPIAs might be due to MWR
- Remember real SPIAs are easier to manage (no repurchasing)

Improvement in portfolio consumption, is the average lifetime increase in consumption when SPIAs are present compared to when they are absent after factoring out Social Security.

The starting portfolio sizes where cherry picked to make the case for the benefits of early annuitization.

I don't know how much I would read into the difference between nominal and real SPIAs. It is likely the result of the differing money's worth ratio slopes. But these slopes were based on real world quotes.

An advantage of real SPIAs over nominal SPIAs is that they don't require repurchasing to keep up with inflation. This is particularly important if the SPIAs are going to be held for a long period of time.

Caveats

- Stock and bond returns, and SPIA prices based on typical times
- Current expectations: lower returns, high SPIA prices
- Should not significantly alter asset allocation or annuitization strategy
- Will effect consumption strategy

- Taxation may affect when to annuitized

- Don't fully understand observed money's worth ratios

- Optimal versus good enough

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Wrapping up, I should probably include a disclaimer.

The analysis I have performed is based on historical data for 1927 through 2013. We are presently in an unusual environment. The P/E ratio for the stock market is relatively high, suggesting lower returns in the future, and bond returns are very low. Since these two are moving in the same direction it is likely that the optimal asset allocation strategy will not change significantly.

The SPIA pricing model I use combines bond prices and mortality data. It will adjust along with the bond interest rate. The optimal annuitization strategy should not change significantly.

The optimal consumption strategy is a different matter. I haven't said much about consumption, but if returns fall, so does the optimal consumption amount.

So far taxes have not been considered. Issues to be considered include tax brackets, separate income and capital gains tax rates, federal and state tax exempt dividends, U.S.-style SPIA taxation, qualified and non-qualified SPIAs, taxation of real SPIAs, and state guarantee association taxes. These factors are complex. U.S.-style SPIA taxation, in which tax preferences expire once the original life expectancy is reached, appears computationally intractable. These factors are so complex, I can’t even say which way they would shift when to annuitize.

Money’s worth ratios slightly greater than 100%, and upward sloping, and downward sloping, money’s worth ratios have been observed. And while plausible explanations have been offered for them, it isn’t clear that they are correct.

The strategies I have computed are the mathematically optimal strategies for the given scenarios. There might be simpler strategies that aren’t optimal, but are good enough for all practical purposes.

Summary

- Floor and upside utility
- Computing the optimal strategy

Wrapping up my talk. I will go over what we have learned.

First I discussed the economics concept of utility, and why it is important. And what a floor and upside utility function looks like. The floor is highly risk averse, while the upside is less risk averse.

In retirement planning you want to maximize expected utility of consumption, not consumption itself, and never maximize wealth.

Then I discussed how by treating the retirement problem as a mathematical problem of optimization under uncertainty, you could use stochastic dynamic programming to compute the optimal strategy. Something which is difficult to do using trial and error.

Optimal Strategy without SPIAs

- Underfunded → stocks (or reduce planned consumption)
- On track → balanced
- Overfunded → stocks

- TIPS should be used when bond allocations are at their highest

We looked at the optimal strategy in the absence of SPIAs.

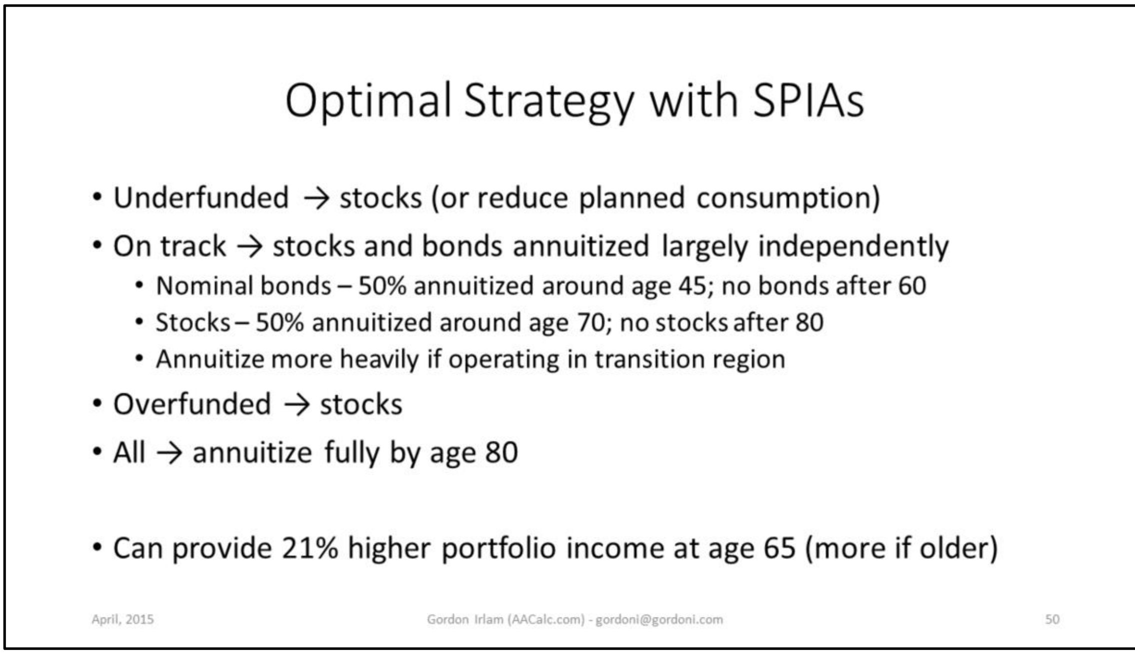
For individuals that are underfunded, in the presence of Social Security, the optimal strategy was 100% stocks.

The optimal asset allocation was balanced for individuals that are on track.

And for the overfunded the optimal strategy was again stocks, this time on account of the lower relative risk aversion.

I will mention that the words underfunded, overfunded, and on track, as used here, have their everyday meanings, not a precise definition.

We also saw, at least for this scenario, TIPS being used at the center of the bond asset allocation band, with nominal bonds used at the periphery.



Lastly the optimal strategy in the presence of SPIAs.

The underfunded and the overfunded should again be 100% stocks.

If you on track, you can think of your total portfolio as a balanced unannuitized portfolio of stocks and bonds to which gradual annuitization is occurring. The mid-point for nominal bond annuitization is around 45. The mid-point for stock annuitization is around 70. The exact ages will depend upon SPIA prices. If you are consuming in the transition region then you should annuitize more heavily than this, possibly even completely annuitizing to lock in consumption.

And everyone, irrespective of wealth, should be completely annuitized by age 80.

The availability of annuities was found to provide a 21% increase in portfolio income for a 65 year old with a cherry-picked $400k of assets.

Questions

- Q&A

- For more information
  - Academic papers - ssrn.com/author=2306779
  - Asset Allocation Calculator (no SPIAs) - AACalc.com
  - Email - gordoni@gordoni.com

I will now take any questions.